Gathering Agents on a Lattice by Coupling Reaction-Diffusion and Chemotaxis

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Abstract

We address the question as to which are the minimal ingredients to obtain a decentralised gathering of agents that move on a lattice. The agents and their environment are described with a stochastic model inspired from biology: the aggregation of the *Dictyostelium discoideum* cellular slime mold. The environment is an active lattice, which cells transmit information according to a reaction-diffusion mechanism. The agents trigger excitations randomly; they move by following excitation fronts. We show that despite its simplicity this model exhibits interesting properties of self-organisation and allows to achieve decentralised gathering. Moreover, observations show that the system has interesting robustness properties, as being able to resist to the presence of obstacles on the lattice and to resist to the addition of noise on the moves on the agents.

keywords:

decentralised gathering problem ; bio-inspired modelling ; cellular automata ; multi-agent systems ; self-organisation ; reaction-diffusion-chemotaxis ; Dictyostelium discoideum ; phase transitions

foreword:

Animations showing the experiments described in this article can be viewed at: `http://www.loria.fr/~fates/Amybia/expe.html`

1 Introduction

Decentralised Gathering Problem Let us consider the problem where agents are initially randomly scattered on a lattice and have to group to form a compact cluster. The agents, all identical, have no idea of their own position, nor do they have on the position of the other agents. All they can do is to send messages that can be relayed, possibly with errors, by the cells of the lattice. The agents have only a rudimentary level of perception and a limited repertoire of actions: they can perceive the state of only their neighbouring cells and the only actions they can undertake is to move to these cells or change the state of the cell on which they are located.

The main question is to determine what are the minimal ingredients involved to achieve a decentralised gathering with these constraints. More precisely, we wish to determine how a simple model can achieve the decentralised gathering where "simple" means using a small number of states for the propagation of messages and a small number of rules for controlling the motion of the agents.

An early reference that addressed the problem of controlling a swarm of robots to form simple shapes is the paper by Sugihara & Suzuki [18]. They proposed decentralised algorithms to form circular or polygonal shapes. One of their assumptions was that robots could perceive the positions of other robots without any limitation. This hypothesis is of course the most demanding and further works followed where robots were considered a having a limited visibility. For example in ref. [4], authors exhibited an algorithm that could gather robots that are initially in the same "visibility component", *i.e.*, each robot of the component is linked to other robots by a path of "visibility" relationship. Their algorithm uses a simple idea: each robot converges toward the "centre of gravity" and ensures that it does not break the visibility component. The moves of the robots were supposed to be instantaneous, an hypothesis which suppresses the risk that the computation of the robot is based on an outdated perception of the world.

Several versions of the gathering problem were examined. For example, in [13], the problem was modified to demand that the robots, which are considered as points, not only gather in the same area but succeed to be all placed on the *same point*. The robots are considered in an "asynchronous" framework : the delay between their starting point and their end point is explicitly taken into account. The solution proposed relies on the assumption that the robots share a common sense of orientation (x and y axis). This showed that there exists a form of equivalence between the a model with instantaneous moves and no sense of orientation and a model with time-dependent moves and a sense of orientation. We refer to the work of Prencipe [17] for recent theoretical developments on the decentralised gathering problem.

In this paper, the problem of visibility of robots is not fundamental since messages are not exchanged directly between robots but are instead transmitted by the environment on arbitrarily long distances. This of course has advantages and drawbacks, which will be studied in the following of the article.

Other Related Problems   As it is frequently noted, the decentralised gathering problem is related to the Leader Election problem, initially introduced in [15], where all cells are initially in the same state and where the goal is to attain a configuration in which a single cell is in a distinguished state. In our problem, the objective is to form a cluster with the agents initially dispersed in random locations ; it is a form of symmetry breaking where some special location has to be chosen by consensus. By contrast with diffusion-limited aggregation, where fractal clusters are formed by random walks of particles, we want the clusters to be compact and efficiently generated. The *amorphous computing* paradigm [1], which uses dispersion of the agents and the introduction of noise in the system, also relates to our work. In such a context, solving the decentralised gathering problem may be used to aggregate components, which can constitute a first step before making computations.

The next section explains the biological inspiration of the model and draws a quick review of the field of Dictyostelium modelling. Section 3 describes formally the model we use to achieve the decentralised gathering. In Section 4, the environmental layer of the model is studied as a source of interest *per se*. In Section 5, we present experiments to explore how the gathering occurs. In Section 6, we evaluate the robustness of the model by applying various perturbations. Finally, we conclude with a short discussion on the results and on their interpretation in the fields of computer science, biology or robotics.

2   Social Amoebae as a Source of Inspiration

The cellular slime mold *Dictyostelium discoideum* is a fascinating species whose individuals usually live as a mono-cellular organisms but may also transform into a multi-cellular organism when needed. In normal conditions, the cells live as single individuals by eating decaying logs, humus and bacteria (*e.g.*, [7]). They reproduce by simple cellular division (mitosis). However, when the environment becomes depleted of food, a gathering process is triggered and single cells aggregate to form a complex organism that will move and react with coordination of its components. The transformation from a group of individual amoebae into a multi-cellular aggregate is a complex phenomenon that involves different stages. This article takes inspiration from the first stage of the multi-cellular organisation process, the *aggregation* stage, which consists in gathering all the cells in a compact mass called a *mound* (*e.g.*, see [19]).

Observations of *in vitro* experiments show that this aggregation is triggered by the spontaneous emergence of "pacemakers" or "signalling centres" (*e.g.*, [9]). These pacemakers are formed by one or several cells that attract other cells that are located in their vicinity. Once the first pacemakers are formed, they are in an unstable situation: under normal conditions, they struggle against each other and merge until only a few pacemakers remain ; these will attract other cells to them to form a group where cell differentiations will occur. From a quantitative point of view, the order of magnitude of the size of an amoeba is $10\,\mu m$, the size of the aggregates can be up to 10^5 individual cells, the gathering can occur

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on a distance as far as 20 mm [3, 7].

The signalling occurs by transmission of waves, which follow typical evolving reaction-diffusion patterns. The waves are constituted of high-concentration profiles of cyclic adenosine monophosphate (cAMP), an intercellular messenger that serves to guide the moves of the amoebae: this phenomenon is called *chemotaxis* in the biological context and we will use it by analogy to qualify the moves of our virtual agents. The origin of these reaction-diffusion patterns resides in the concomitant realisation of four actions: (a) a cell synthesises cAMP internally until there is enough product to be emitted ; (b) when an amoeba detects a high increase in external cAMP concentration, it follows the concentration gradient (chemotaxis) and releases its own internal cAMP (exocytosis) (c) it then becomes insensitive to cAMP during a given refractory period , (d) in the meanwhile, the cAMP released diffuses and excites other sensitive cells, etc.

It is out of scope of this article to review the models that have been proposed to study the dynamics of *Dictyostelium*. Interested readers should refer to the works by Nagano [16] or by Deutsch & Dormann [8] as entry points to the literature. Problem solving by simulation of excitable media by reaction-diffusion models is studied in [2]. Our proposition is to take the essential ingredients of the aggregation mechanism of *Dictyostelium* to achieve the decentralised gathering of agents. The model is described by simple rules that couple reaction-diffusion and chemotaxis. These laws are stochastic and use three probabilities ; we show that for particular settings of these probabilities, the model provides a solution to the problem of achieving a quick and robust decentralised gathering on a lattice.

3 Coupling Reaction-Diffusion and Chemotaxis

The Reaction-Diffusion-Chemotaxis scheme we study is here presented as a stochastic discrete dynamical system where time, space and state are discrete. It is described at two levels: the description of this scheme is given as a set of `instructions` followed by *one particular* formalisation of these instructions with a mathematical description. It should be noted that other models and other descriptions of the scheme are possible; the study of how small changes in the model's description affect its behaviour is left for future work. The formalisation we present is meant to be as simple as possible; it thus provides a tool to examine *sufficient* conditions under which decentralised gathering is possible. Two layers compose it: the *environmental* layer is a cellular automaton that models a reaction-diffusion process while the *particle* layer describes the moves of virtual amoebae (or simply *amoebae* in the following). The notations we use to present the model are summarised in Table 1.

Space is modelled by a regular lattice $\mathcal{L} = \{1, \ldots, X\} \times \{1, \ldots, Y\}$ in which each cell $c = (c_x, c_y) \in \mathcal{L}$ is associated to a state. For a time t, we denote by σ_c^t the state of a cell c and by P_c^t the number of amoebae it contains. The formalisation of the Reaction-Diffusion-Chemotaxis scheme consists in expressing σ_c^{t+1} and P_c^{t+1} as a function of $(\sigma_c^t)_{c \in \mathcal{L}}$ and $(P_c^t)_{c \in \mathcal{L}}$. In this paper, we arbitrarily use

Table 1: Notations used in the description of the model

$(p_\mathrm{T}, p_\mathrm{E}, p_\mathrm{A})$	.... transmission rate, emission rate, agitation rate
$\mathcal{B}, \mathcal{R}$	.... random operations
$c \in \mathcal{L}$	.... cell of the grid
σ_c^t	.... state of c at time t
P_c^t	.... population of c at time t
N_c/E_c^t	.... cells / excited cells in the neighb. of c at time t
$\tilde{N}_c^t/\tilde{E}_c^t$	free cells / excited free cells in the neighb. of c at time t

the eight-cell neighbourhood, *i.e.*: $N_c = \{c' \in \mathcal{L}, |c'_x - c_x| = 1 \text{ or } |c'_y - c_y| = 1\}$. Note that it is not strictly equivalent to the Moore neighbourhood as cells are excluded from their own neighbourhood, and as cells in the border of the lattice $\mathcal{L}$ have a smaller neighbourhood.

3.1 The Environmental Layer

Figure 1: Transition rule of the environmental layer (simple reaction-diffusion)

The set of possible states for each cell is $\{0, \ldots, M\}$: the state 0 is the *neutral* state, the state M is the *excited* state, the states 1 to $M - 1$ are the *refractory states*. We will call neutral, excited, or refractory, a cell of a given configuration that is in the neutral, excited, or refractory state, respectively. The evolution of a cell of the environment is represented on Figure 1 ; it is described with the following rules:

- A neutral cell which has at least one excited neighbour becomes excited with probability p_T; otherwise it stays neutral. p_T is called the *transmission rate*.

- An excited cell becomes refractory in one step.

- A refractory cell decrements its state by 1 until it becomes neutral.

To express these rules formally, for a time t, let E_c^t be the set of excited cells in N_c, the neighbourhood of c: $E_c^t = \{c' \in N_c \,|\, \sigma_{c'}^t = M\}$. We denote by $\mathrm{card}\{X\}$ the cardinal of a set X and by $\mathcal{B}(\alpha)$ the Bernoulli random variable of parameter α, *i.e.*, a random variable that equals 1 with probability α and equals 0 with probability $1 - \alpha$. The local rule governing the evolution of each cell is:

$$\sigma_c^{t+1} = \begin{cases} M & \text{if } \sigma_c^t = 0 \text{ and } \mathrm{card}\{E_c^t\} > 0 \text{ and } \mathcal{B}(p_\mathrm{T}) = 1 \quad (R1) \\ \sigma_c^t - 1 & \text{if } \sigma_c^t \in \{1, \ldots, M\} \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (R2) \\ 0 & \text{otherwise} \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (R3) \end{cases}$$

3.2 The Amoebae Layer

The amoebae are supposed to be all identical, and in constant number as no birth or death process is considered. The movement of amoebae obeys the following rule: only one amoebae is allowed to move from a source cell to a target cell. No limitation is put on how many amoebae may move simultaneously to the same target cell; however, a cell that contains two (or more) amoebae will not accept further amoebae. These rules are designed as a trade off between the need to limit the number of amoebae per cell (to observe clusters) and the need to keep the cells updating rules simple (more elaborate procedures can be designed to manage simultaneous moves to a given cell).

We define an *empty* cell as a cell which contains no amoeba and a *free* cell as a cell that contains less than *two* amoebae. Informally, the motion rules state that, at each time step, for each non-empty cell, one single amoeba obeys:

- With probability p_A, move to a neighbour free cell randomly. p_A is called the *agitation rate*.

- Move randomly to an neighbour free *excited cell*.

- Stay on the same cell.

For $t \in \mathbb{N}$ and $c \in \mathcal{L}$, let $\tilde{N}_c^t$ and $\tilde{E}_c^t$ be the set of *free* cells and *excited free* cells, respectively, in the neighbourhood of c. For a finite set X, we denote by $\mathcal{R}(X)$ the random variable that selects one element in X with uniform probability; with the convention $\mathcal{R}(\emptyset) = \emptyset$. $\mathcal{R}$ is used to select a random neighbour for moving. To represent the move of one amoeba from a non-empty source cell c to a target cell Δ_c^t, adopting the convention that $\Delta_c^t = \emptyset$ if no move occurs, we write:

$$
\begin{aligned}
&\text{if } \mathcal{B}(p_\mathrm{A}) = 1 \text{ then} \quad \Delta_c^t = \mathcal{R}[\tilde{N}_c^t] && (R4)\\
&\qquad\quad \text{else} \quad \text{if } \sigma_c^t = 0 \text{ and } \mathrm{card}\{\tilde{E}_c^t\} > 0 \text{ then } \Delta_c^t = \mathcal{R}[\tilde{E}_c^t] && (R5)\\
&\qquad\qquad\quad \text{else } \Delta_c^t = \emptyset && (R6)
\end{aligned}
$$

The number of amoebae in a cell c is updated following:

$$
P_c^{t+1} = P_c^t + \mathrm{card}\{c' \in \mathcal{L} \,|\, \Delta_{c'}^t = c\} - \mathrm{card}\{\Delta_c^t\}
$$

There are of course many other ways to formulate these rules. From a programming point of view, what is most important to note is : (a) we adopted a cellular-automaton point of view rather than a multi-agent one, *i.e.*, actions are cell-centred, amoebae are represented by an attribute of cells rather by an independent list of agents ; (b) As our formulation does not forbid simultaneous moves of amoebae to the same cell, up to nine amoebae are allowed to share the same cell in the unlikely event where all the neighbours of a cell move *simultaneously* to a target cell with *one* amoeba on it. (c) the noise rule is the first in priority ; this means that for high values of noise, attraction effects are likely to be destroyed by this noise. Weaker versions of noise are possible, for example by moving randomly only if no excitation is perceived.

3.3 Coupling the Environment and the Amoebae

How do amoebae and the environment interact ? The interaction is given with the single law :

- A non-empty *neutral* cell becomes excited with probability p_E, the *emission rate*.

For the sake of simplicity, we deliberately formulate this excitation rule by considering only the difference between empty and non-empty cells. This cell-centred point of view is meant for facilitating the coding of the model and, if needed, its implementation on massively parallel devices. We can of course consider other models where excitations are triggered by each amoeba independently.

Formally, for $t \in \mathbb{N}$ and $c \in \mathcal{L}$; the interaction between amoebae and the environment is modelled by:

$$\sigma_c^{t+1} = M \quad \text{if } \sigma_c^t = 0 \text{ and } P_c^t > 0 \text{ and } \mathcal{B}(p_\mathrm{E}) = 1 \quad (R7)$$

As rules R1 and R7 may interfere, we need to clear this ambiguity of formulation. The are several ways to do this, we choose here to combine rule R1 and R7 into a single rule R1':

$$\sigma_c^{t+1} = M \quad \text{if } \mathcal{B}(p_\mathrm{T}) = 1 \text{ and } \sigma_c^t = 0$$
$$\text{and } \operatorname{card}\{E_c^t\} > 0 \text{ or } [P_c^t > 0 \text{ and } \mathcal{B}'(p_\mathrm{E}) = 1] \quad (R1')$$

The rule R1' states that a cell becomes excited only if it is neutral, with probability p_T. The excitation is either received from a neighbouring cell or, with probability p_E, from the amoebae it contains.

To finish the presentation of the model, as our goal is to study the *simplest* model in terms of states and rules, we set the excitation level to $M = 2$; the set of states is thus $\{0, 1, 2\}$.

4 Experimental Study of the Environment

We now study the qualitative behaviour of the environmental layer by means of simulations. Readers familiar with reaction-diffusion models and with second-order phase transitions may jump to the next section where we examine the gathering phenomenon.

4.1 The Fully Deterministic Case is Static

Experiment. First, we examine the system in the fully deterministic case: the transmission rate and the emission rate are set to 1, the agitation rate is 0: $(p_\mathrm{T}, p_\mathrm{E}, p_\mathrm{A}) = (1, 1, 0)$. Figure 2 shows the evolution of a system that contains only three amoebae on a small lattice size $(30, 20)$. We observe that the amoebae are the source of excitation waves that propagate at the speed of one cell per unit of time.
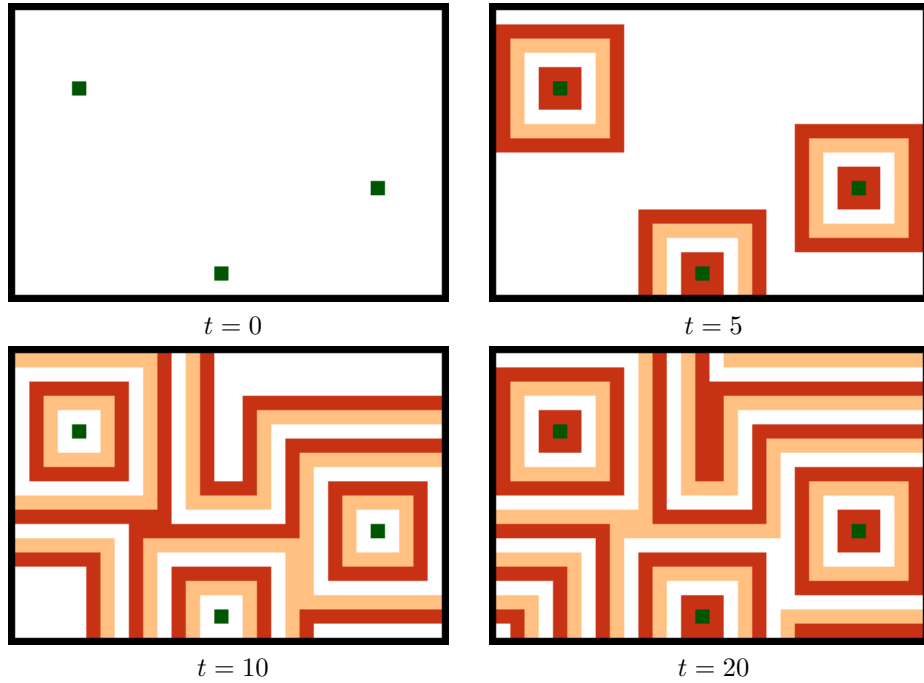
Figure 2: Four views of the evolution of the model with two amoebae in the fully deterministic case for a grid size $(30, 20)$ and $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (1, 1, 0)$. Amoebae are represented by black squares, white squares are neutral cells, darkest brown/grey squares are excited cells, lighter brown/grey squares are refractory cells. This colour code is kept in the following. All simulations are made with the *FiatLux* CA-simulator [11].
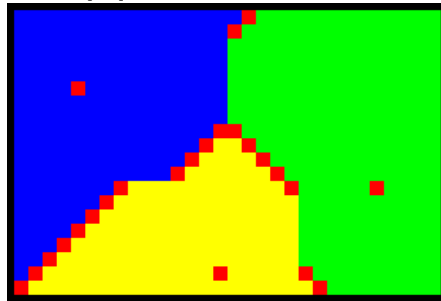

Figure 3: The corresponding regions of influence for each amoebae (see text). Diagonal frontiers are special cells that belong to two or three influence regions.

We call a set of adjacent cells that are all in the excited state M an *excitation front*. Excitations fronts propagate at the speed of one cell per unit of time. We observe that when two excitations fronts meet, they simply annihilate. As a result, the gathering is impossible: no chemotaxis ever occurs as excitation fronts do not hit the amoebae.

Interpretation. Contrarily to classical diffusion waves, it is a well-known phenomenon that reaction-diffusion fronts annihilate when they meet. This property is respected by the model. It implies that the transfer of information may be limited to a certain part of the lattice. Let us define informally an *influence region* of an emitting cell as the set of cells that will receive the excitation wave emitted by this cell. Intuitively, we see that in the fully deterministic case, the influence regions of the amoebae correspond to the discrete Voronoi diagram of the lattice with amoebae as centre points (see Fig. 3 for an illustration and [2] for a more detailed analysis). It is important to note that communication between amoebae via the environment can not occur in the fully deterministic case. Indirect communication happens when some amoebae enter the influence region of other amoebae.

4.2   Non-coherent Regime

Experiment. To observe the effects of transmission errors in the environment, we set the transmission rate to $p_T = 0.99$, *i.e.*, we introduce a 1% chance that a cell fails to receive an excitation from its neighbours. Figure 4 shows an evolution of the system for these settings. We observe that for small simulation times the system behaves qualitatively as for the non-perturbed case: waves are initiated by amoebae, they propagate until they collide and annihilate. However, as transmission errors accumulate, the waves progressively loose their coherent shape. Additional sources of waves appear : these are spiral waves whose behaviour is well-studied in reaction-diffusion media, whether discrete or continuous [ref.]. As time advances, more and more of these persistent spiral waves appear. Finally, when the coherence is totally lost, amoebae start moving erratically as they sometime receive excitation fronts.

Interpretation. Small transmission errors in the environment create situations where excitation waves survive arbitrarily long periods of time. The multiplication of such waves causes confusion in the system and does not allow the amoebae to group into clusters. We call this behaviour where waves develop independently of the position of the amoebae, the *non-coherent regime*.

4.3   The Extinction Regime

For smaller values of p_T the loss of coherence is observed even more rapidly. Interestingly enough, when the transmission rate was small enough, we observed that the waves were no longer persistent. We call this new qualitative behaviour, where waves spontaneously disappear, the *extinction regime*. We experimentally
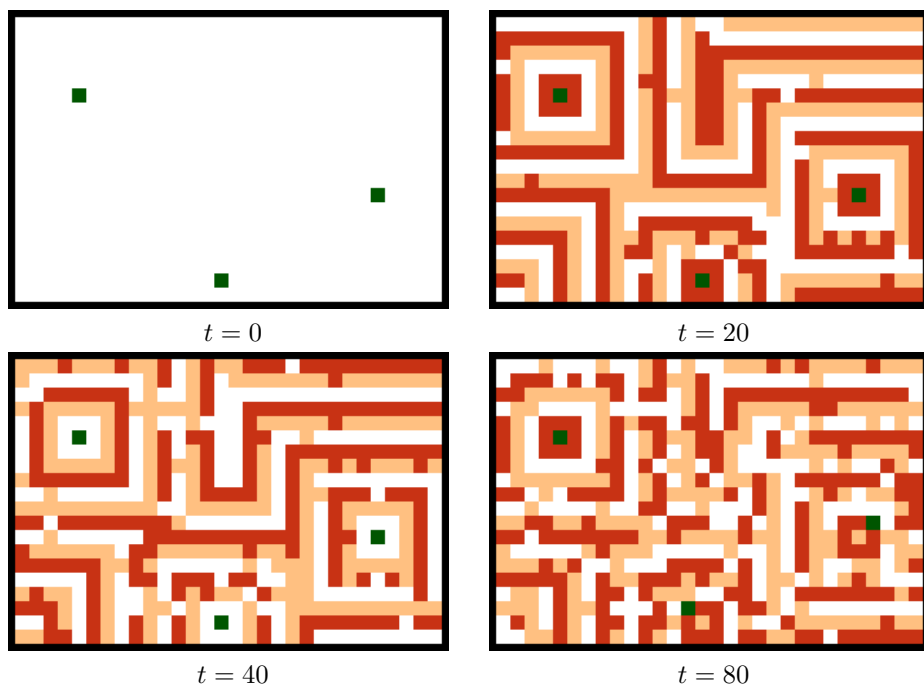
Figure 4: Loss of coherence occuring with non-perfect transmission rate $(p_{\mathrm{T}}, p_{\mathrm{E}}, p_{\mathrm{A}}) = (0.99, 1, 0)$ on a small lattice of $(30, 20)$.
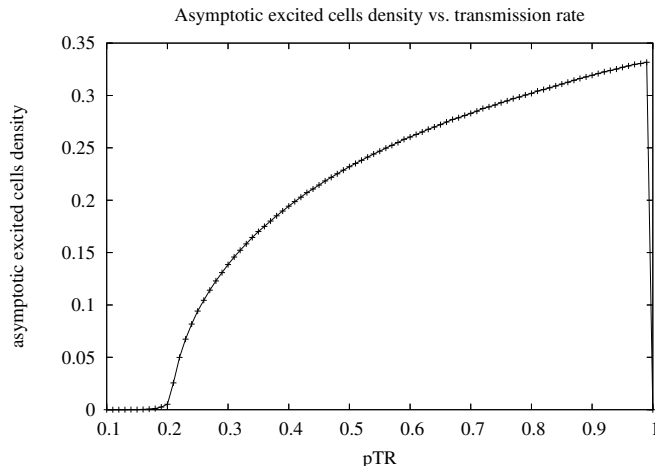
Figure 5: Estimated asymptotic density of excited states as a function of p_T (see text). Lattice size is $(100, 100)$, the environment is depleted of amoebae, 10% of the cells were intially set to the excited state.

observed that the transition from the non-coherent regime to the extinction regime is sharp and occurs for $p_T \sim 0.20$.

Experiment. In order to understand the origin of this abrupt change of behaviour, we considered a system depleted of amoebae, where some cells were randomly set to the *excited* state with probability 10% and left *neutral* otherwise. To separate the non-coherent regime from the extinct regime, we monitored the evolution of the density of excited cells: $e(t) = \text{card}\{c \in \mathcal{L} \,|\, \sigma_c^t = M\}/X.Y$

We expect this quantity to reach quickly zero for the extinction regime and to remain strictly positive for the non-coherent regime. To test this hypothesis, we varied the transmission rate p_T by 1% steps from 0.01 to 1, for a lattice size $(100, 100)$, and measured the evolution of the $e(t)$ during $10\,000$ time steps. We computed an approximation of the asymptotic density of excited cell by measuring the average value of $e(t)$ for $t \in [5\,000, 10\,000]$. Figure 5 shows this average as a function of p_T. It confirms the presence of a qualitative change for $p_T \sim 0.20$.

Interpretation. The shape of the curve suggests that the transition between the two regimes is a second-order phase transition that occurs for $p_T \sim 0.20$. We can also note that for p_T close to 1, we observe a non-regular behaviour that is due to finite-size effects: because of the limitation of the lattice, the probability of apparition of persistent waves is small and the non-coherent regime may not be reached. This means that for values of p_T close to 1, the system oscillates between the static regime described in section 4.1 and the non-coherent regime described in the previous section.

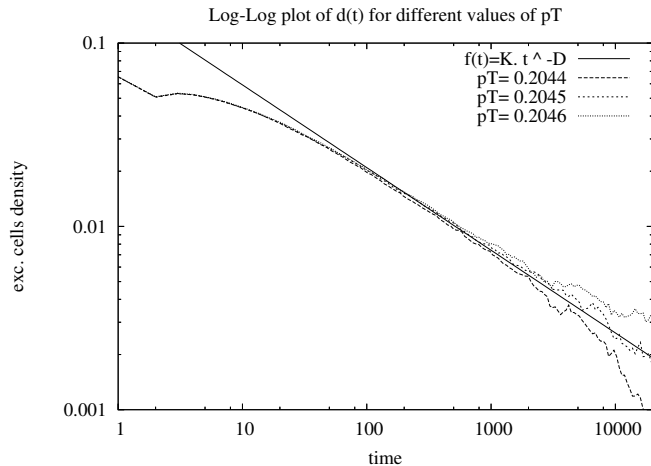**Refining the previous experiment.** To analyse the nature of the change

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Log-Log plot of d(t) for different values of pT

f(t)=K. t ^ -D ———
pT= 0.2044 ----------
pT= 0.2045 ··········
pT= 0.2046 ··········

Figure 6: Evolution of the density of excited cells $e(t)$ around the critical value
transmission rate p_T (log-log scale). Lattice size is $(400, 400)$. The straight line
has slope $-\delta_\mathrm{DP} = -0.451$, which is predicted by the directed percolation theory.

near criticality, we monitored the evolution of $e(t)$ for the transmission rates
$p_\mathrm{T} \in \{0.2044, 0.2045, 0.2046\}$. For different lattices sizes, up to $(400, 400)$, the
average value of $e(t)$ was measured for 50 random initial samples. We observed
that this small variation of p_T, of the order of 10^{-4}, separated the extinction
regime from the non-coherent regime (in which excitations survive arbitrary
long periods of time).

By plotting $e(t)$ in a log-log scale, it appeared that the curve closely followed
a power-law for $t \gtrsim 100$. It is well-known in statistical physics that the power-
laws observed in phase transitions are not arbitrary: there are particular sets
of exponents that characterise the evolution of the system near criticality. The
class of different models that can be described by the same sets of exponents is
called a *universality class*. By analogy with previous observations made in asyn-
chronous cellular automata, we tested whether the phase transition belonged to
the universality class of *directed percolation* [5, 10]. The evolution of excited
cells density should then follow $e(t) \sim t^{-\delta}$ near criticality, with $\delta = 0.451$ for
two-dimensional lattices (this number is known only experimentally [14]). Fig. 6
shows the evolution of the average value of $e(t)$ for a lattice size $(400, 400)$; we
see that, as expected, for $p_\mathrm{T} = 0.2190$, the curve of $e(t)$ closely follows a straight
line in a log-log plot, with a slope close to the predicted value.

Interpretation. Experiments suggest that the universality class of the phase
transition from the non-coherent regime to the excitation regime is directed
percolation, as it was observed for asynchronous Elementary Cellular Automata
and in other CA models (see [10] for an overview). This calls for further studies
on the origin of phase transitions in cellular systems and on how these phase
transitions may bu used to gain a better control on the gathering process.

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Synthesis. The existence of different regimes in the environmental layer underscores a strong condition for the amoebae to achieve the gathering: the medium which implements the reaction-diffusion has to relay the excitations without error. Experimentally, we observe that the system becomes more robust as the value of M is increased, *i.e.*, for $p_\mathrm{T} \to 1$ the probability of falling in the non-coherent regime becomes smaller for large values of M. We believe that this question is strongly related to the metastability problem in the Greenberg-Hastings model studied in [12]. The study of this robustness is a problem that arises from these observations and is left for further studies. In the following, we will consider that the environmental layer is perfect by taking $p_\mathrm{T} = 1$.

5   Obtaining the Decentralised Gathering

We now examining the gathering behaviour both from a qualitative and quantitative point of view. We now examine the effect of changing Our method consists in varying the probability of emission p_E, beginning our study by simulating a system on a small lattice and then examining it on a large lattice to see what are the scaling properties of the gathering behaviour.

5.1   Gathering in Clusters and Pacemaker Effect

Experiment. To illustrate the effect of changing the emission rate, we examined the behaviour of the system for the values $p_\mathrm{E} \in \{0.01, 0.10, 0.50, 0.80\}$. To initialise the system, we assigned to each cell a 10% probability to contain an amoeba. Unless otherwise mentioned, we keep this random initial condition for all the following experiments ; this is compatible with experiments conducted by other authors in the biological modelling context (*e.g.*, [3]).

We experimentally observed that the gathering of amoebae occurs for all the values of p_E considered. Moreover, the smaller p_E was, the quicker the gathering occurred. Figure 7 shows the evolution of the system for $p_\mathrm{E} = 0.10$: a compact cluster emerges in a few hundred steps ; it then emits waves with a good regularity. To quantify the gathering of amoebae, we propose to examine the temporal evolution of the *bounding box ratio* (BBR), defined as the ratio of the surface of the largest rectangle containing all the amoebae over the total surface of the lattice. Formally, for a time t, we denote by $C = \{c \,|\, P_c^t > 0\}$ the set of non-empty cells. Let $x_\mathrm{min}, x_\mathrm{max}$ (respectively $y_\mathrm{min}, y_\mathrm{max}$) be the minimal and maximal value of c_x (respectively c_y) such that $c = (c_x, c_y) \in C$, then we define:
$$\mathrm{BBR} = \frac{(x_\mathrm{max} - x_\mathrm{min}).(y_\mathrm{max} - y_{min})}{X.Y}$$

This parameter is rather simplistic since it captures only a small part of the system's organisation into clusters. Note that it is a "strong" criterion, in the sense that it requires that no single amoeba should be forgotten from the gathering.
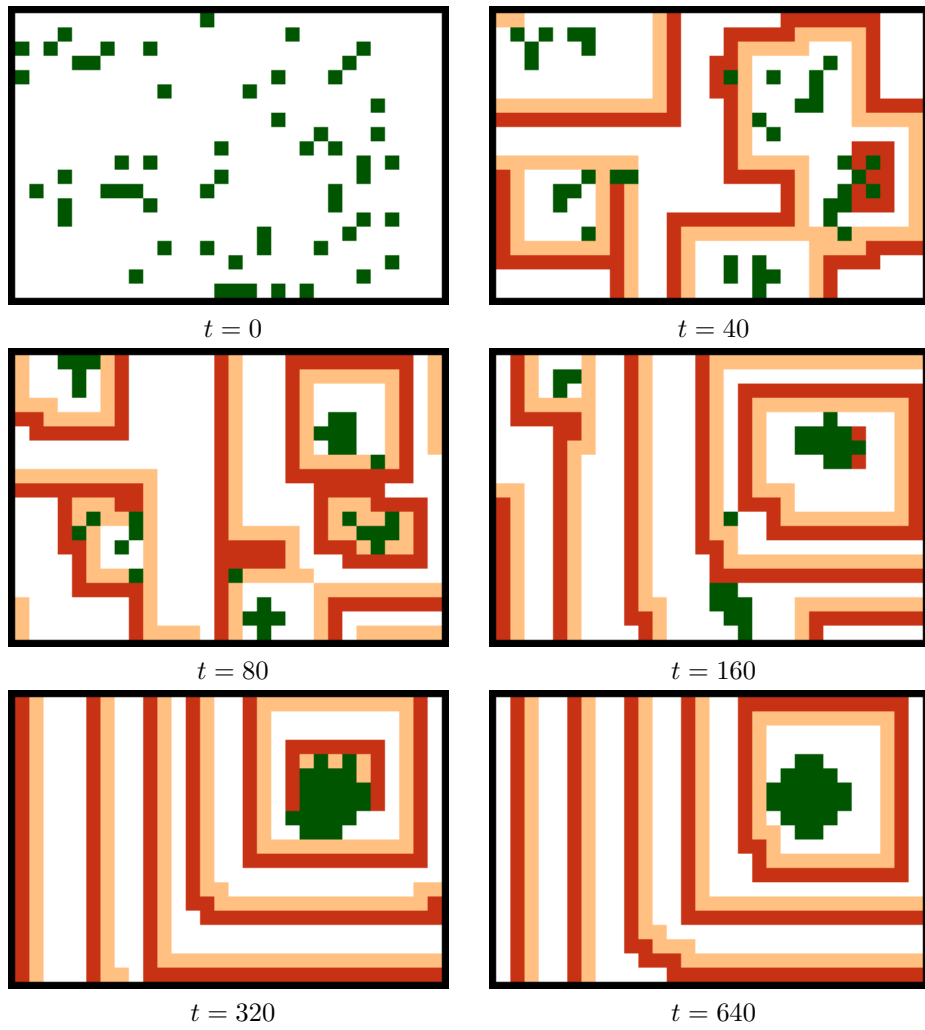
Figure 7: Sequence showing the formation of a pacemaker with $(p_\mathrm{T}, p_\mathrm{E}, p_\mathrm{A}) = (1, 0.10, 0)$ and $(X, Y) = (30, 20)$.
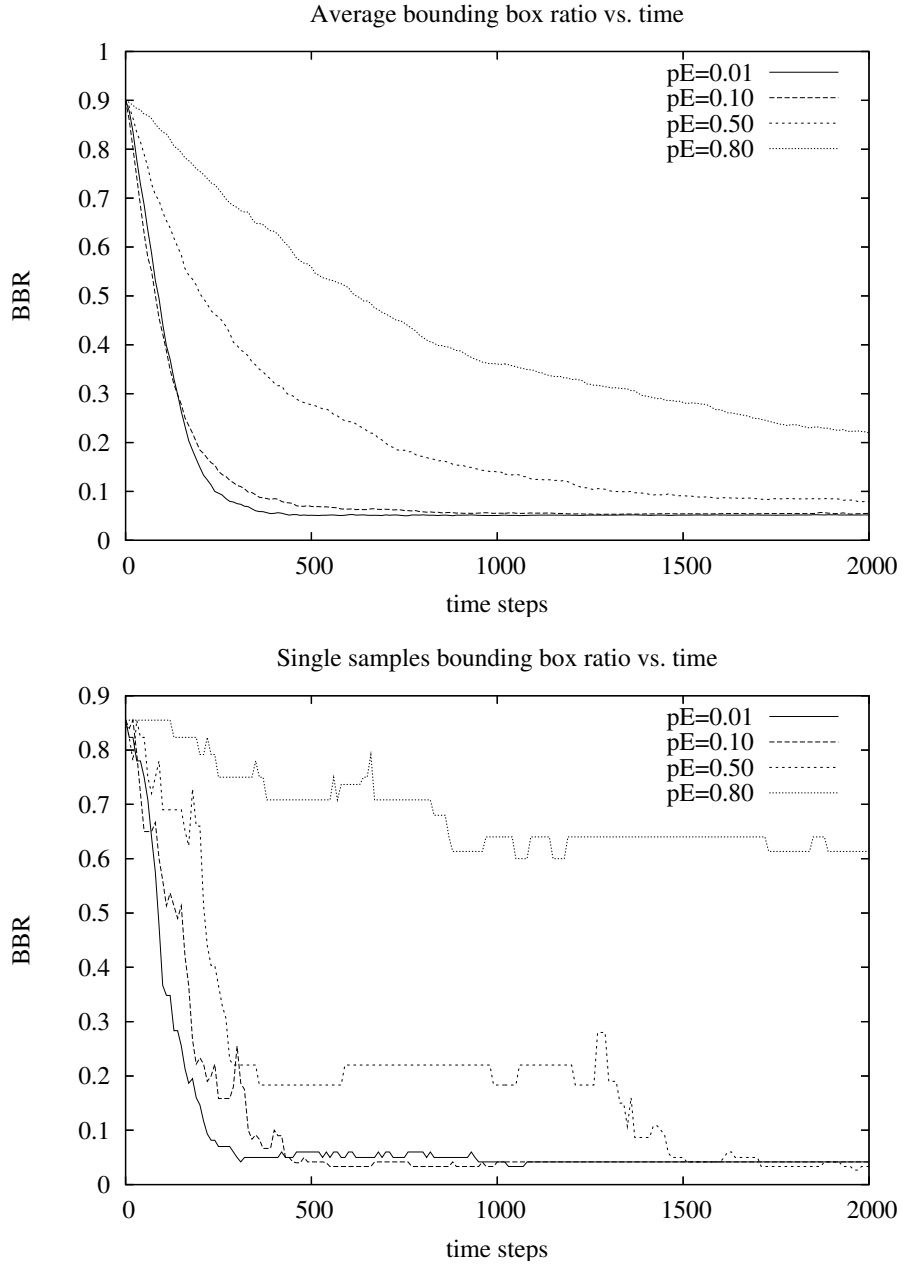
Figure 8: Evolution of the bounding box rate (BBR) as a function of time for a grid size $(30, 20)$, $p_\mathrm{T} = 1$, $p_\mathrm{A} = 0$, and different values of p_E. Average evolution for 50 samples (top) Example of a single sample evolution (bottom).

Figure 8 shows evolution of the BBR for different values of p_E. The first plot (top) displays the average evolution of for 50 independent samples. We see that this evolution is regular; statistical measures confirm the counter-intuitive fact that the gathering process is accelerated when p_E is decreased. However, for very small values of the emission rate ($p_E < 10^{-3}$), the gathering process is slowed as the waves are not emitted regularly. It is an open problem to determine the value optimal value of p_E as a function of the grid size.

However, note that the smoothness of the curves comes from the averaging and does not well describe the evolution of a single sample. The bottom plot of Fig. 8 shows the evolution of a single samples for three values of p_E. The curve obtained for $p_E = 0.5$ shows three stages which correspond to:

- for $t \in [0, \sim 300]$ amoebae gather to form two unstable clusters,

- for $t \in [\sim 300, \sim 1200]$ the two clusters emit excitations fronts with an irregular pace, during this stage, influence regions of the two clusters shrink or extend due to the irregularities of emissions,

- for $t \gtrsim 1200$ one of the two clusters, the "winning" cluster, have its influence region touch the other cluster and "captures"' its amoebae.

Interpretation. For small lattices, the decrease of p_E to values as small as 10^{-2} allows the system to achieve the decentralised gathering in a few thousand steps. The gathering is initiated when randomly chosen amoebae emit an excitation wave which will attract non-emitting amoebae to it. The repetition of the process creates more populated regions of the lattice. These dense regions, the *clusters*, examined as a whole, emit excitation waves and attract the cells that are in their influence region. The more a cluster increases in size, the more regularly it emits excitations. This correlation between the size of a cluster and its frequency of emission creates a positive feedback ; asymptotically, all cells should gather in the same cluster. When a cluster has a reasonable size, it emits waves with a good regularity ; by analogy with the biological phenomenon, we call this the *pacemaker* effect (*e.g.*, [9]).

5.2 Gathering on Larger Lattices

How does this self-organising system behaves for larger lattices and for a great number of amoebae? We now examine how the gathering process is structured, how several cluster merge to form one single cluster.

Experiment. Keeping $p_T = 1$ and $p_A = 0$, for a lattice size $(150, 100)$ and for an observation time of $20\,000$ time steps, we measured the evolution of the BBR with $p_E \in \{0.01, 0.10, 0.50, 0.80\}$. Fig. 9 shows one evolution of the system for these settings. We repeated this experiment with 100 random samples ; using the BBR quantification of the gathering, we observed that the system always achieved to stabilise to a small value (below 15%) which corresponds to the formation of a single cluster and thus to the achievement of the decentralised
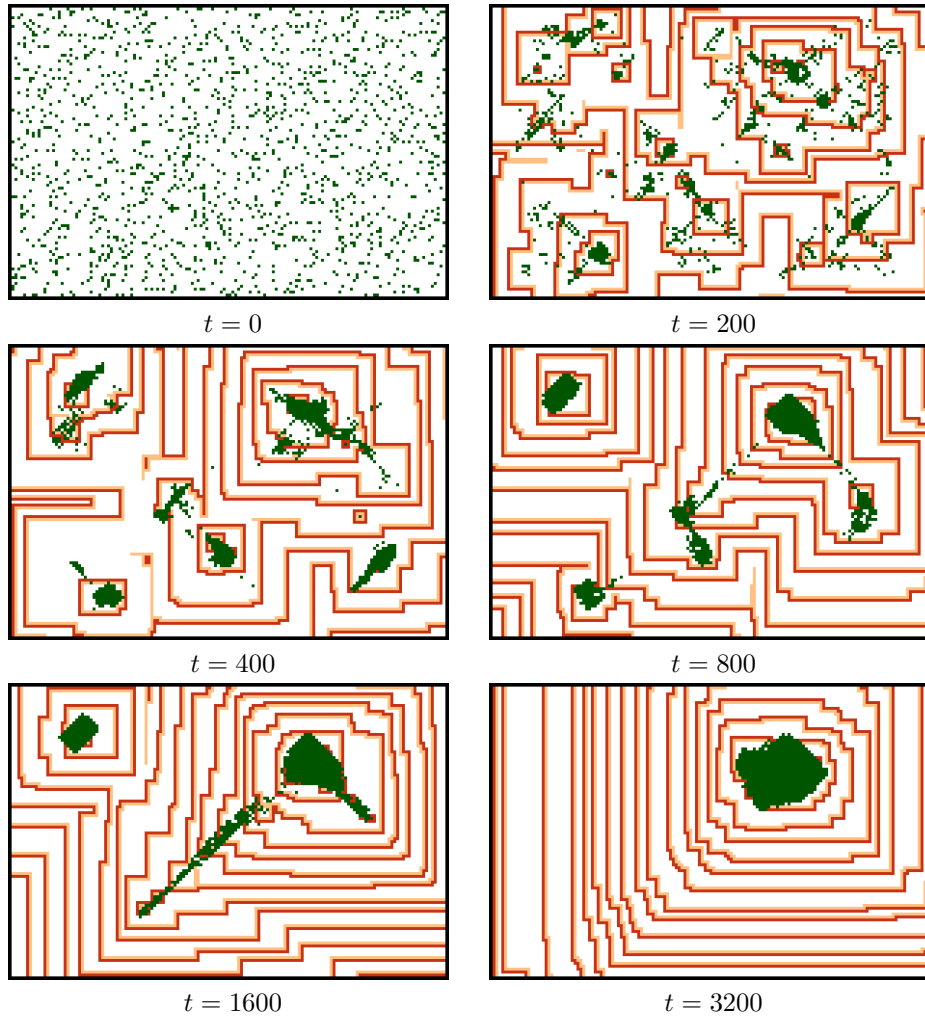
Figure 9: Evolution of the model with perfect transmission rate and no agitation: $(p_\text{T}, p_\text{E}, p_\text{A}) = (1, 0.01, 0)$.

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gathering task. Statistical data is displayed on Fig. 11 p. 20 for $p_E = 0.01$ and $p_E = 0.10$ (no noise curves).

Moreover, observing some simulations, we noticed surprising effects that can only be observed on large lattices. For example, it happened that clusters were destroyed and spontaneously reformed when, by "chance", the attracting cluster shrank its influence region. We also observed rare cases where a cluster divided into two parts, each part being attracted by a different cluster.

Interpretation. The self-organisation of amoebae in clusters is a phenomenon that emerges from the competition between clusters. When two clusters compete for extending their influence region, we observe that the cluster that contains more amoebae has a tendency to win. Indeed, if, in a given period of time, a cluster A emits a wave and a cluster B does not emit a wave, as excitation fronts travel at constant speed, the influence region of cluster A will extend and the influence region of cluster B will shrink. From the observer point of view, frontiers between influence regions perform a biased random walk where the bias is favourable to the cluster with the highest emitting frequency. Once a cluster sees its influence region totally shrunk, its amoebae come under the influence of another cluster and they move towards the attracting pacemaker by forming streams. Noteworthy is the analogy with the biological phenomenon where these streams are also observed, at least in *in vitro* experiments.

6 Robustness to Perturbations

Now that we have established that the gathering is possible by decreasing the value of p_E, we examine how the model resists to two types of perturbations: randomness imposed on the moves of the amoebae (p_A) and introduction of obstacles on the lattice. This robustness study is particularly important if we are to apply the reaction-diffusion-chemotaxis scheme to control robots aggregation.

6.1 Self-organisation with Agitation

Experiment. In order to examine whether the system is robust to noise superimposed in the moves of the amoebae, we set the agitation rate to 20%, keeping the two other parameters unchanged: $(p_T, p_E, p_A) = (1, 0.01, 0.20)$. Figure 10 shows one evolution of the system for these settings and Fig. 11 shows statistical measures obtained for different values of p_E and p_A. It is remarkable that the addition of $p_A = 20\%$ of random move does not slow much the gathering process. For higher values of p_A the perturbations are too important to allow compact and stable clusters to form ; however, a form of gathering can still be observed, at least for p_A smaller than 0.5.

Interpretation. The gathering still occurs even when the moves of the amoebae are perturbed by a small amount of noise. The most noticeable difference in the dynamics is that groups of small size do not appear. This tends to show that there exists a link between the stability of groups of a given size and the
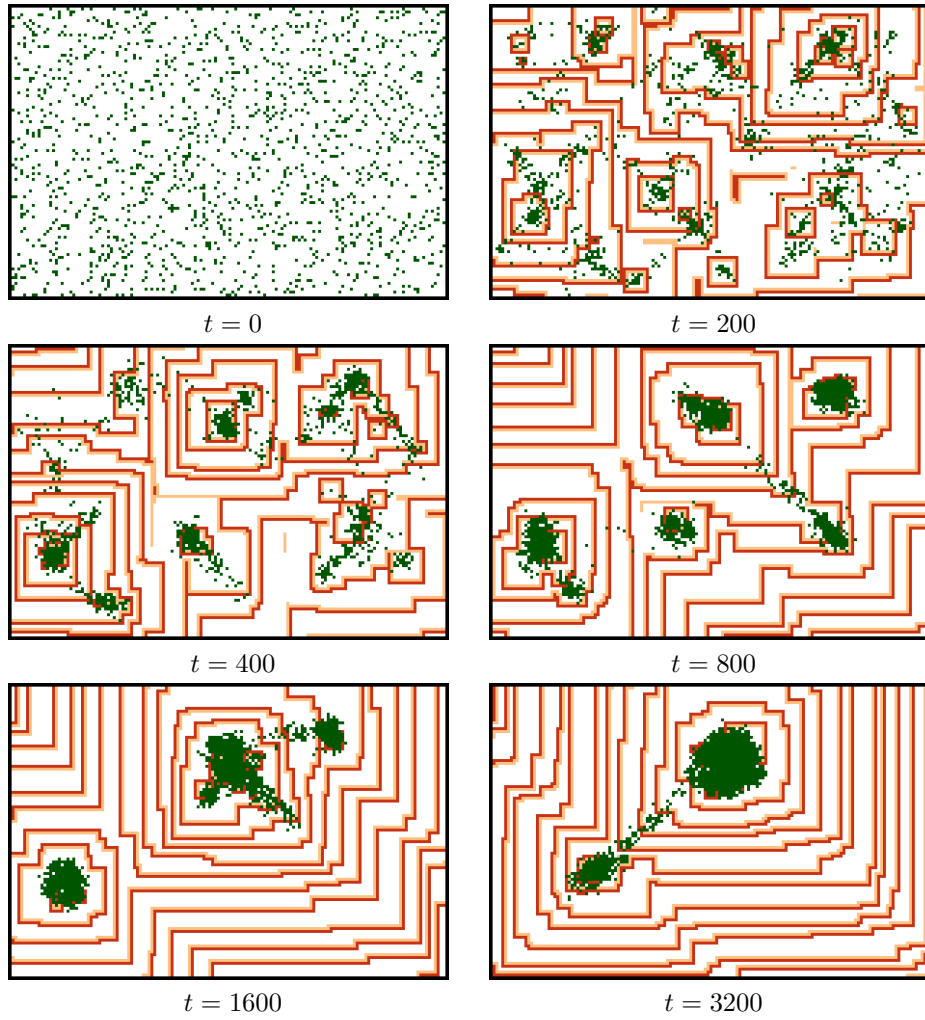
Figure 10: Evolution of the model with perfect transmission rate and small agitation: $(p_\text{T}, p_\text{E}, p_\text{A}) = (1, 0.01, 0.1)$.
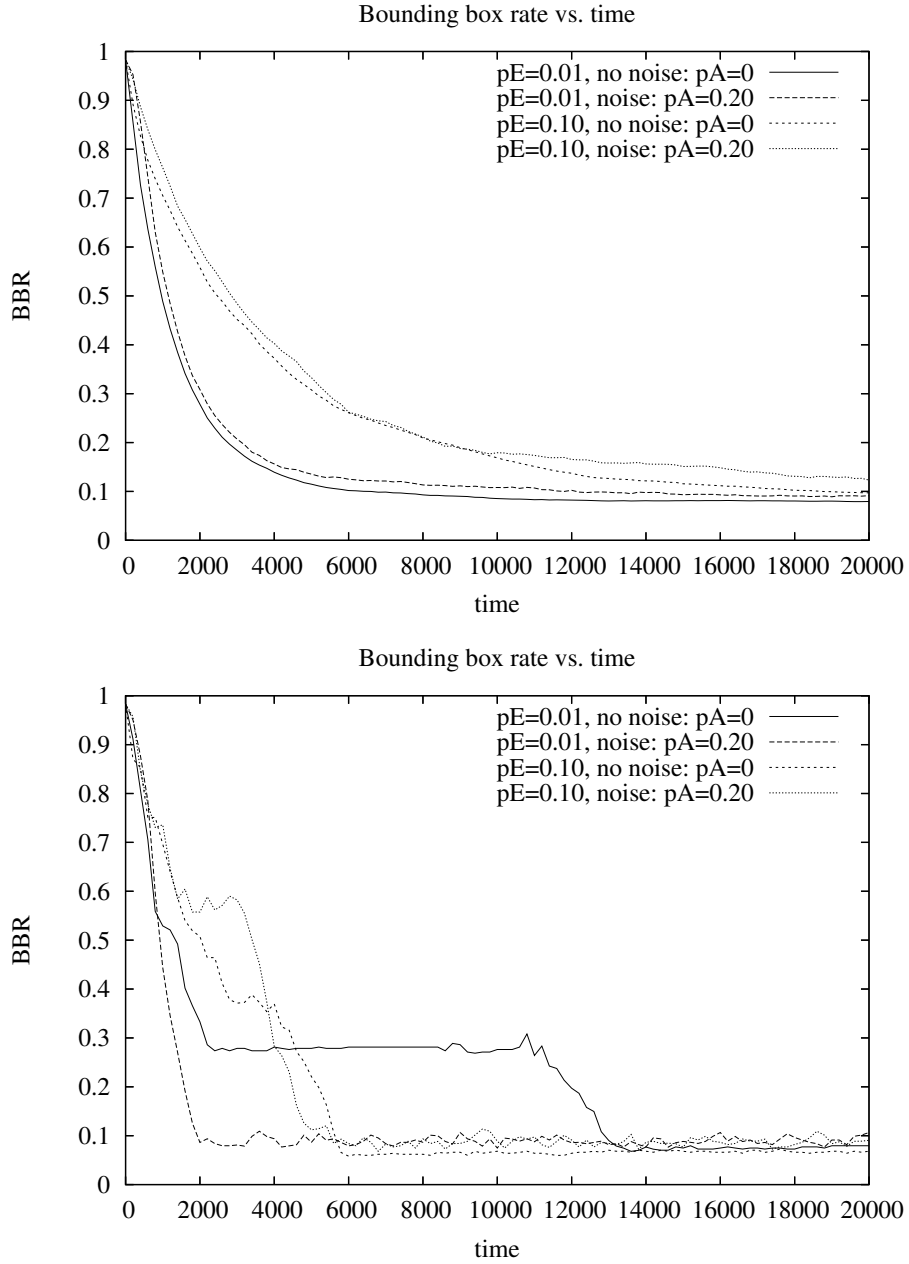
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Figure 11: Evolution of the BBR as a function of time for a grid size $(150, 100)$, $p_{\mathrm{T}} = 1$, and different values of p_{E} and p_{A}. Average evolution for 100 samples (top) Example of a single sample evolution (bottom).

Table 2: Synthetic view of the qualitative behaviours oberved by simulating the aggregation model

p_E	p_T	p_A	qualitative behaviour
1	1	0	static
any	$[0.2, 1[$	0	non-coherent
any	< 0.2	0	extinct
0.10	1	0	self-organising (slow)
0.01	1	0	self-organising (quick)
0.01	1	0.2	self-organising (quick)

parameter p_A. Finding a relationship between the minimal group size (if such a property exists) and the quantity of noise is another interesting question that arises from these observations.

6.2   Obstacles on the Lattice

Experiment. What happens to the system when the topology of the lattice is modified? In particular, how do amoebae operate when obstacles are introduced? Real amoebae evolve in very inhomogeneous media and also need to achieve the gathering despite the presence of numerous obstacles. In Fig. 12, we present the evolution of the system where obstacles (straight lines) are placed randomly on the lattice. Obstacles do not allow information nor amoebae to cross. The corresponding statistical data is displayed on Fig. 11.

Interpretation. We observe that the system is robust to the presence of obstacles. In the parts that are totally disconnected from the rest of the lattice, isolated groups are formed. In the others parts of the lattice, the aggregation process is not perturbed by the presence of obstacles. In particular, in some parts of the lattice, it is possible to observe the amoebae taking narrow spaces to converge to a pacemaker. In this case, the streams formed are thin and they usually split and re-form several times before reaching a pacemaker. We underline that this type of robustness is somewhat obtained "for free": it can be seen as a an emergent property since at no time it was explicitly coded in the local rules governing the agents and the environment.

7   Discussion

Synthesis   This article proposed an original solution to the decentralised gathering problem in the case where simplistic agents or robots have to group by moving on a lattice. This solution uses the reaction-diffusion-chemotaxis scheme, a simple discrete dynamical system inspired from a biological example (the aggregation phenomenon of *Dictyostelium*). The rules that govern the system are simple: only three states are needed for propagating messages in the lattice and only three rules guide the virtual amoebae actions. The description
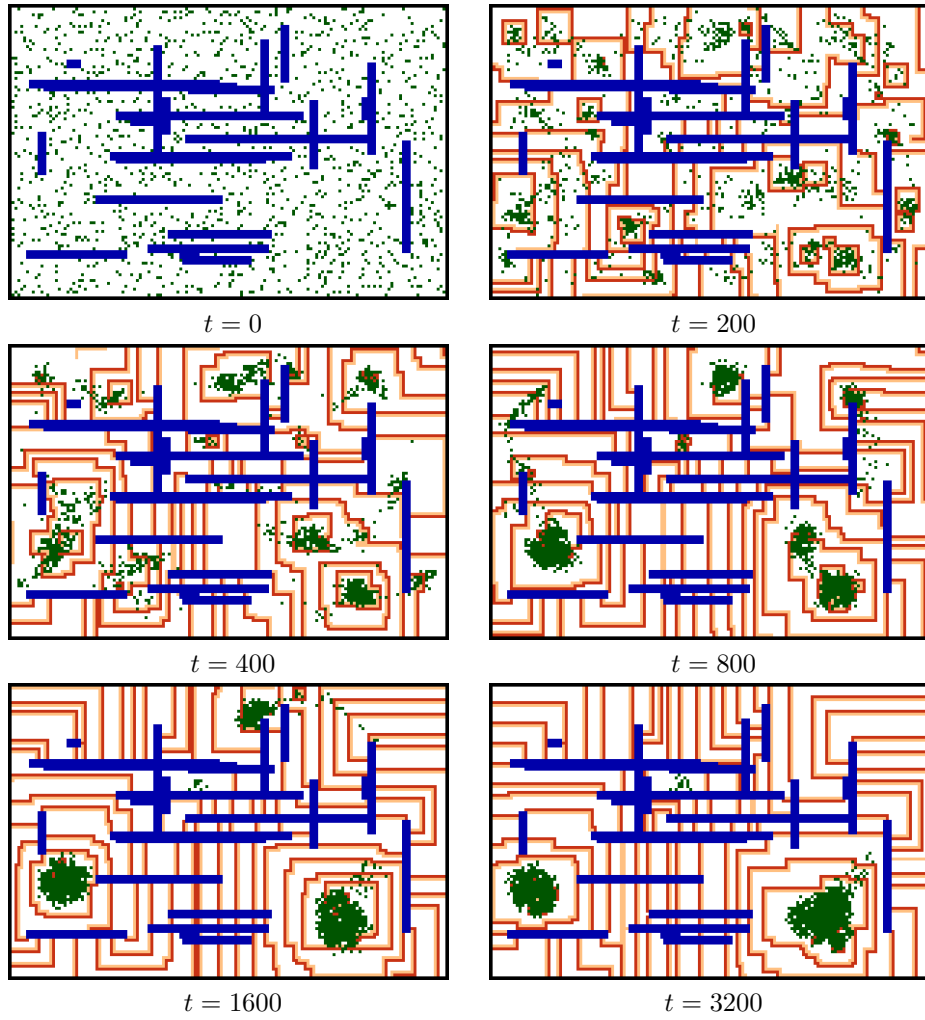
Figure 12: Evolution of the model with obstacles, perfect transmission rate and small agitation: $(p_\mathrm{T}, p_\mathrm{E}, p_\mathrm{A}) = (1, 0.01, 0.1)$.

of the scheme uses two layers : the environment simulates a simple reaction-diffusion mechanism to propagate information ; the agents follow a chemotaxis law to move towards other agents and form clusters. One single parameter, the emission rate p_E, controls the interaction between the agents and their environment. The experiments showed that this parameter was easy to tune and allowed to observe the gathering for a wide range of values. However, it should not be too low, otherwise excitation waves are not frequently produced, and not too high, otherwise excitation waves annihilate and do not reach the virtual amoebae. To give an image, all happens as if the agents had to find the right compromise between "speaking" and "hearing".

A Hierarchical Process Our study put an emphasis on the exploration of the qualitative behaviour of the dynamical system. Four different qualitative behaviours were exhibited: the static regime, the non-coherent regime, the extinction regime and, last but not least, the self-organised regime (see Table 2). The self-organising regime was examined under various conditions. We showed that the dynamics of the process, far from trivial, allowed the system to achieve the gathering even when the agents' motion was subject to a high level of noise. The dynamics was presented to the reader with an "ascendant" view, *i.e.*, from the local rule to the global behaviour. Conversely, we may also look at the gathering phenomenon with a top-down view:

1. The gathering phenomenon results from a competition between clusters; the bigger a cluster, the higher its probability to "capture" other clusters.

2. A cluster captures another cluster when its "influence region" touches it.

3. The extension or shrinking of the influence regions depends on the average frequency of emission of the "pacemakers" of each cluster,

4. The "pacemaker" effect of a cluster results from independent emissions of excitation waves and the diffusion of this waves out of the cluster.

5. Excitation waves propagate at a constant speed without attenuation

6. Each cell of the lattice follows a simple local rule to update its state and to move the amoebae it contains to the neighbouring cells.

Among interesting properties observed in the system, it was shown that the gathering could also occur in the presence of obstacles on the lattice. The gathering process was not much perturbed as the virtual amoebae could take advantage of narrow corridors to find their way to a pacemaker.

Perspectives in Computer Science It is now necessary to compare our model to other bio-inspired models that use virtual chemotaxis such as virtual ants. Virtual ants use simple diffusion and chemotaxis to realise complex tasks in a decentralised way. Virtual ants were used to solve various complex problems such as the Travelling Salesman Problem (see [6]). However, to our knowledge,

the use of simple diffusion and chemotaxis is delicate. In particular, a technical part is required to tune the diffusion and evaporation coefficients in order for an agent not to be trapped by its own emission of pheromones. Furthermore, the intensity of pheromones decreases exponentially in space, this can be an advantage in certain cases but can also limit the aggregation process on large distances.

Are there other problems where the reaction-diffusion-chemotaxis scheme would prove to be efficient? We may for example use the phase transition observed in the environmental layer to design an artificial decentralised means of consensus. For example, one could use similar mechanisms to achieving a self-diagnosis task : we can imagine that a network of interconnected sensors decides to change its behaviour when a critical fraction of defective components is reached.

Perspectives in Robotics   If we apply this Reaction-Diffusion-Chemotaxis scheme to group robots, its main advantage is the simplicity to program the robots: all they need to do is to follow the excitations in the right direction (from neutral to excited). These excitations can take many forms, for example of a light that would be emitted by small components regularly placed on the space where robots move. Note that as we do not use any memory in the robots, the system is naturally self-stabilising: an unwanted displacement of robots or a failure to emit an excitation should not perturb the gathering phenomenon. As we saw in Section 4, the main limitation of the model is the non-robustness of the environmental layer: messages have to be relayed synchronously and without any error in order to prevent the creation of self-entertained excitation waves. How to make the environment more robust without complicating too much the local rules? This challenge is left open for future research.

Back to biological modelling   To conclude, let us also consider going back to biological modelling. Clearly, there is a significant difference between our model and the real amoebae: the reaction-diffusion process is not implemented by the environment but by the amoebae themselves (see Section 2). Despite of this difference, our model reproduces the qualitative behaviour of the competition between the pacemakers to form an aggregate. It also shows that the formation of streams of amoebae does not necessitate much communication between cells but might be a mere consequence of the global dynamics of the system.

We also observed that the propagation of excitation waves in the environment was subject to a phase transition. This may give us a hint on how real amoebae collectively decide to start the gathering phase. So one might also take this model a starting point to examine whether phase transitions are a mechanism to obtain consensus in a societies of simple organisms.

8 Acknowledgements

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